PHILOSOPHY TRIPOS Part II

Monday 5 June 2017 09.00 – 12.00

Paper 7

MATHEMATICAL LOGIC

Answer **three** *questions only.*

Write the number of the question at the beginning of each answer.

STATIONERY REQUIREMENTS
20 Page Answer book x 1
Rough Work Pad

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

1. Sketch a proof that any axiomatisable extension of (first order) **ZFC** is incomplete if it is consistent.

2. Consider the following recursive definition of a truth predicate for the language of basic arithmetic:

    Atomic A is true iff A is provable in **PA**
    A∧B is true iff A is true and B is true
    ¬A is true iff A is not true
    ∀xA is true iff, for each numeral **n**, A[x/**n**] is true

    Given that every recursive predicate can be captured in **PA**, is this a basis for a proof that **PA** captures its own truth predicate?

3. Assuming that **PA** is ω-consistent, show how to construct a formula G such that **PA**+G is consistent but ω-inconsistent.

4. Show that a first-order theory either has only finite models or is not categorical. Show that there is a categorical second-order theory. What, if anything, is the philosophical significance of these facts?

5. Prove the compactness theorem for first-order logic with identity. Use it to show that the set of all first-order arithmetical truths has non-isomorphic models.

6. Is there any way to rescue Hilbert's Programme from the problems arising from the incompleteness theorems?

7. Prove in **ZFC** that if |A|≤|B| and |B|≤|A| then |A|=|B|.

8. Define cardinal addition, multiplication and exponentiation. Show the following, where A, B and C are disjoint sets:

    (i) $|A|^{\mathbf{0}} = \mathbf{1}$
    (ii) $|A|^{\mathbf{1}} = |A|$
    (iii) $|A|^{|B|} \times |A|^{|C|} = |A|^{|B|+|C|}$
    (iv) $(|A|^{|B|})^{|C|} = |A|^{|B|\times|C|}$
    (v) $|A| < |\wp A|$ and $|\wp A| = 2^{|A|}$

9. Are there good reasons to believe or disbelieve the axiom of choice?

10. Does the Löwenheim-Skolem theorem imply that we can do without postulating uncountable sets?

END OF PAPER