Ramsey's transcendental argument

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One of the papers of Ramsey's *Nachlass* which his widow Lettice sold to the Hillman library at Pittsburgh is a set of notes entitled 'The infinite'. Embedded in these notes is the following curious argument for the axiom of infinity:

> We can say that the idea of infinity proves its existence. (Wittgenstein's extra prop). But the sign ∞ proves nothing. We can prove it this way. It is clear that there may be an ∞ of atoms and whether there are or not is an empirical fact, and this possibility implies an ∞ of objects, as it were to be the possible atoms. In this way it is clear that transcendentally taken the axiom of infinity is true, though empirically it is doubtful.

The argument occurs again in a slightly more finished piece of writing entitled 'The number of things in the world'. But nothing like it is to be found in anything Ramsey himself actually published, and although both the pieces just mentioned are included in Galavotti's (1991) selection of papers from the *Nachlass*, the argument just quoted seems to have been ignored except for a brief discussion in my own book (2000). Yet it does seem to me to be a very interesting argument. So I want here to return to it in rather more detail than I had space for in my book and try to answer four questions about it:

1) What is the context of Ramsey's argument?
2) Why did Ramsey not publish it?
3) When did Ramsey think of it?
4) Does it have any merit independent of Ramsey's own views?

I

Let us begin, then, by getting clear about the argument itself. What is clear straightaway is that the context Ramsey intends is the system of the *Tractatus*, in which he had been immersed since he prepared the first draft of its English translation early in 1922. So we cannot hope to understand Ramsey's argument without first going some way into this context. Now it would be a brave man who confidently asserted what the key idea of the *Tractatus* is. (Certainly what Wittgenstein himself calls his key idea – that logical constants do not refer – is rather hard to present in a way that makes it anything like the lynchpin of the book.) But it is at any rate *one* of the key ideas of the *Tractatus* that the task of a proposition is not merely to say how things stand in the world but to contrast the way they do stand with other ways they could have stood but don't. The job of a proposition, that is to say, is to carve up the ways things might stand into two classes: the proposition is then true or false according as the way things are is in one or other of these two classes. And Wittgenstein takes it that these different ways things might stand – possible worlds, to use the modern jargon – must have something in common, in order that they should be different ways *our* world could be rather than just wholly distinct worlds with nothing whatever to do with one another. The elements which different possible worlds have in common Wittgenstein calls *objects*. 'Object' is thus for Wittgenstein a technical term, referring to whatever it is that our language presupposes in order that it should be significant.

We should grant one thing straightaway: it is hard to be confident that the existence of objects really does flow from the key idea just alluded to. Certainly the existence of objects is famously the first of the claims of the *Tractatus* that Wittgenstein himself publicly renounced, and the only argument he ever offers for believing it – the argument for substance of *Tractatus* 2.0211-2 – is notoriously brief and problematic. Nonetheless we must grant Wittgenstein's claim for the time being if we are to be in a position to appreciate Ramsey's argument, since it is a claim which Ramsey simply presupposes.

And if we do grant Wittgenstein's claim, it is easy to see that a great deal follows. It follows at once, for instance, that objects are necessarily existent, just because they are by definition what different possible worlds have in common. (And presumably, therefore, most of the humdrum things we knock against in our mundane lives are not objects in Wittgenstein's technical sense, since it seems we can quite coherently express the possibility of their non-existence.) It follows, too, that there is no genuine relation of identity that can hold or fail to hold between objects: what changes in the transition between possible worlds is how objects are combined with one another to form atomic facts; what the objects are does not change, because they are the hinges about which the possibilities turn and hence are constant.

But although there are according to the *Tractatus* no *genuine* identity statements, there are many propositions that are *apparently* of this form. What appear to us to be meaningful identity statements linking proper names actually involve disguised descriptions to be analysed by means of Russell's famous re-writing device. But at this point there is a difficulty. Russell analyses g(the f) as

$$(\exists x):fx.(y)fy\supset x=y.gx,$$

which still contains the symbol for identity. So if, as Wittgenstein claims, there is no relation of identity, the analysis is still strictly meaningless. Wittgenstein's solution is to adopt a new convention for interpreting quantified variables: where one variable occurs in the scope of another, Wittgenstein assumes (unlike Russell) that the ranges of interpretation of the two variables do not overlap. Consider, for example, $(\exists x,y)xRy$: for Russell this means that something is R-related to something; whereas for Wittgenstein it means that something is R-related to something *else*. This elegant notational device allows Wittgenstein to re-express the Russellian analysis of g(the f) as

$$(\exists x).fx.gx.\sim(\exists x,y).fx.fy.$$

(In words: something is both f and g, and there are not two things which are both f.)

Notice also that since which objects there are does not vary between worlds, how *many* there are does not vary either. And if how many objects there are does not vary between worlds, there cannot be a genuine proposition which expresses how many there are. The best we could do in this regard would be to say something which *presupposes* for its significance that there are a certain number of objects. If we did that, we would show but not say how many things there are.

Now you might very well think that what I have just said is wrong and that there is in fact a way of saying how many things there are: you might think indeed that Wittgenstein's *own* notation for avoiding the identity sign allows us to express exactly this. If fx is any propositional function,

$$(\exists x_1, \ldots ,x_n).fx_1. \ldots .fx_n$$

apparently says that there are at least n things that are f. So if Tx is some propositional function which is true of every object of a certain sort (e.g. $fx \vee \sim fx$), and if we let

$$p_n =_{\mathrm{Df}} (\exists x_1, \ldots, x_n).Tx_1. \ldots .Tx_n,$$

then p_n seems to say just that there are at least n things.

Seems to, but does not. What has always to be borne in mind is that Wittgenstein's way of reading nested quantifiers is a *device*. We can invent all manner of such devices – all manner of combinations of signs – but whether any such combination succeeds in saying something significant depends on whether it carves up the ways the world could be into two classes, those in which it is true and those in which it is false. (This, remember, was part of what I earlier called Wittgenstein's big idea.) And if there are not enough objects, his device will not say something false but will simply not say anything at all. Suppose, for instance, that there are only three objects a, b and c. Then p_2 says the same as $Ta.Tb \vee Tb.Tc \vee Tc.Ta$, which is a tautology, and p_3 says the same as $Ta.Tb.Tc$, which is also a tautology. But what does p_4 say? Nothing remotely similar to the preceding sentences is available. So we are forced to conclude that p_4, despite appearances, is not a proposition at all but merely a jumble of signs without significance. More generally, the pattern is this. If there are N things in the world, the sequence

$$p_1, p_2, \ldots$$

starts with N ways $p_1, p_2, \ldots, p_N$ of expressing tautology; but from then onwards the signs p_{N+1}, etc., rather than being, as we previously thought, ways of expressing something false, are in fact ways of expressing nothing at all.

Do not be too hard on yourself if you made the mistake, though: Wittgenstein says nothing in the *Tractatus* to guard against it, and when Ramsey went to visit Wittgenstein in Austria in September 1923, he evidently persuaded Wittgenstein how easy a mistake it is to make. (It may indeed be that Ramsey himself had made it.) For in Ramsey's copy of the *Tractatus*, at the point where the text explains that one cannot say 'There are 100 objects' or 'There are $\aleph_0$ objects', Wittgenstein added an extra proposition intended to clear up the confusion (see Lewy 1967):

> The proposition 'there are n things such that …' presupposes for its *significance*, what we try to assert by saying 'there are n things'.

We may be sure, then, that even if Ramsey did not understand the point before he went to Austria, he certainly did when he returned to Cambridge to being his first term of study as a graduate student in October 1923.

This, then, is the Tractarian background. With it in place, Ramsey's argument is quickly explained. Let $q_{\aleph_0}$ be the claim that there are infinitely many *empirical* things (electrons, protons, or whatever). This may well, as a matter of fact, be false. But what is clear, Ramsey thinks, is that it is significant. And if it is significant, the sentence $p_{\aleph_0}$ is also significant. But, as we have seen, the signs

$$p_1, p_2, \ldots, p_{\aleph_0}$$

are all either tautological or meaningless. So in particular $p_{\aleph_0}$ cannot be significant without being true. Since it is significant, therefore, it is true. But $p_{\aleph_0}$ is just the axiom of infinity. (Or, more strictly, it shows what the axiom of infinity tries illegitimately, to say.) So we may conclude that the axiom of infinity is true.

II

'The number of things in the world' is not a finished article ready for publication: it launches into its subject far too abruptly to be that. Nonetheless, it is quite close to being in a suitable form for inclusion as a section in a longer article. Yet Ramsey never published it. Why not?

There could be any number of mundane reasons for this, of course, but what I want to show here is that Ramsey's paper 'The foundations of mathematics', which he published in 1926, contains the clues to a particularly straightforward explanation for his abandonment of the transcendental argument. That paper is nowadays famous principally (and for many readers, I suspect, only) for the distinction Ramsey draws between the set-theoretic and the semantic paradoxes. This distinction enables him to argue that a simple theory of types suffices to solve the set-theoretic paradoxes, leaving the semantic paradoxes to be solved at the level of meaning, with the advantage that the simple theory of types has no need of Russell's problematic Axiom of Reducibility.

But it is actually a little strange that this is nowadays seen as Ramsey's principal achievement in the philosophy of mathematics, since the idea is not really his: the distinction between two types of paradox had already been made by Peano, as Ramsey knew, and the observation concerning the simple theory of types which he drew from it is not in itself especially deep.

But 'The Foundations of mathematics' does contain another big idea, and it is this *other* idea that led him to abandon his transcendental argument. We have already seen that Wittgenstein's notation allows us to form the string of signs

$$p_n =_{\mathrm{Df}} (\exists x_1, \dots, x_n).Tx_1. \dots .Tx_n,$$

which seems to say that there are at least n things but if there are not n things is actually meaningless. Already in 'The number of things in the world' Ramsey notes that this lurking possibility that we are talking gibberish is very inconvenient, since 'in making complicated signs, if we are not careful,' we shall involve such forms as this. He then argues that it would be far more convenient if we could give the string of signs a meaning and suggests that 'the most suitable meaning to give it is that of contradiction'. (1991, p.172) But this does not yet overturn Ramsey's transcendental argument because, as he observes, $p_2.{\sim}p_3$, for instance, is 'not really the expression of a proposition "There are exactly two things", and yet it is possible to treat it symbolically exactly as if it was.' Thus Ramsey's position at this time remains that p_2 'has no meaning (unless we define it arbitrarily to mean contradiction)' except in the case in which there are at least two things.

But what happened, and provides a sufficient explanation for Ramsey's abandonment of his transcendental argument, was that he adopted a new notation which allowed him, as he thought, to define *non-arbitrarily* a sequence of propositions which switches from tautology not to meaninglessness but to contradiction. To explain Ramsey's new idea we need to recall one more item from the theory of quantification in the *Tractatus*. One of the ways envisaged there of forming quantified expressions is to take a proposition p and replace some name 'a' in it with a variable x. The result is an instance of what Ramsey calls a *predicative function*. It is a symbolic notation whose role is to pick out a certain class of propositions, viz. all those which are just like p except that they may have in place of 'a' some other name of the same type. As Ramsey points out in

‘The foundations of mathematics’, if *fx* is a predicative function, there is a clear sense in which *fa* says the same about *a* as *fb* says about *b*. The fact which obtains if *fa* is true has just the same structure as the fact which obtains if *fb* is true: the only difference is that the latter fact has *b* in it where the former has *a*. What Ramsey did was to introduce a quite different notation for picking out a class of propositions. A *propositional function in extension* (Ramsey, 1931, p. 52) is a notation $\varphi_e x$ such that, for any name ‘*a*’ of the appropriate type, $\varphi_e a$ expresses a proposition involving *a*. There is no longer any requirement that $\varphi_e a$ should say about *a* the same as $\varphi_e b$ says about *b*.

What matters here is that this notion of propositional function in extension enables Ramsey to define a propositional function

$$T(x,y) =_{\mathrm{Df}} (\varphi)\varphi_e x \equiv \varphi_e y$$

with the property that *T(a,a)* is a tautology and *T(a,b)* is a contradiction for any two distinct objects *a* and *b*. He can then define p_2 to be the logical sum of all the propositions of the form $\sim T(x,y)$. Similarly p_n can now be defined to be the logical sum of all propositions of the form $\sim T(x_1,x_2).\sim T(x_2,x_3).\ \ldots\ .\sim T(x_{n-1},x_n)$. And $p_{\aleph_0}$ is the logical product of the propositions p_n for all finite *n*. The result of all this is that with these new definitions Ramsey’s sequence

$$p_1, p_2, \ldots, p_{\aleph_0}, \ldots$$

goes from tautology not to meaninglessness but to contradiction, thus pulling the rug from under Ramsey’s argument.

III

Before we go on let us, as Ramsey would say, look around and see where we have got to. Ramsey’s transcendental argument is as follows:

1. If $p_{\aleph_0}$ is meaningful, it is true.
2. If $q_{\aleph_0}$ is meaningful, $p_{\aleph_0}$ is meaningful.
3. $q_{\aleph_0}$ is meaningful.

So $p_{\aleph_0}$ is true.

As we have seen, it is a sufficient explanation for Ramsey’s abandonment of the argument that he adopted a notion – that of propositional functions in extension – which makes premise 1 false.

For Ramsey the point of the notion of propositional function in extension was that it was central to his attempt to show that mathematics consists of tautologies. His difficulty, however, was that although the notion was essential to his project, it vitiated his argument for something else that was essential, namely the axiom of infinity. He was therefore reduced to offering, in the last paragraph of the essay as published, what is little more than a rhetorical flourish:

> The Axiom of Infinity ... if it is a tautology, cannot be proved, but must be taken as a primitive proposition. And this is the course which we must adopt, unless we prefer the view that all analysis is self-contradictory and meaningless. (1931, p.61)

This is not satisfactory, and Ramsey knew it. About the same time as he was correcting the proofs of ‘The foundations of mathematics’, Ramsey delivered a paper to the British Association in Oxford in which he admitted that ‘there still remains an important point in which the ... theory must be regarded as unsatisfactory, and that is in connection with the Axiom of Infinity.' (1931, p.78)

Ramsey's failure to find an argument for the Axiom of Infinity that is cotenable with his theory of propositional functions in extension is thus not peripheral but a mortal blow to his version of logicism.

But can we date Ramsey's argument? I think that we can, but to do so we need to continue the narrative a little beyond Ramsey's return from his first meeting with Wittgenstein in September 1923.[1]

It is clear that Ramsey started on the work which became his famous article on 'The foundations of mathematics' soon after he had returned from Austria. He had told his mother in a letter written during the visit that he would 'try to pump [Wittgenstein] for ideas for its further development which I shall attempt', and this seems to be just what happened. We know that he was preoccupied for some time with issues arising from the Wittgensteinian notation for identity. In November 1923 he wrote to Wittgenstein (McGuinness 1995, p. 191) about what he thought was a difficulty of expressing 'Something other than *a* is *f*'. Wittgenstein wrote back immediately with the answer

$fa.\supset.(\exists x,y).fx.fy:\sim fa\supset(\exists x)fx$,

and Ramsey had to admit (McGuinness 1995, p.194) that he had not fully understood the notation.

By the time Ramsey was ready to depart for his second visit to Austria in March 1924, he had made a breakthrough. In a letter to Moore in February of that year[2] asking for a reference in support of his application for an Allen Scholarship, Ramsey reported that

> I have got on W[ittgenstein]'s principles a new theory of types without any doubtful axiom which gives all the results of Russell's one, and solves all the contradictions.

What Ramsey is referring to here, of course, is his use of Peano's distinction between types of paradox to argue for a simple theory of types without the need for reducibility. Ramsey must therefore have come upon this argument quite early in his graduate work.

But he can hardly at this point have come upon his other big idea, propositional functions in extension. For he goes on to tell Moore that

> Wittgenstein and I think it wrong to suppose with R[ussell] that mathematics is more complicated formal logic (tautologies); and I am trying to make definite the vague ideas we have of what it does consist of.

And the whole point of the notion of a propositional function in extension in 'The foundations of mathematics' is that it is what Ramsey uses to *show* that mathematics consists of tautologies.

'The foundations of mathematics' did not appear in print until late in 1926, but almost all the work that went into it was done much earlier. After six months in Vienna, (during which he had to field various complaints from his over-anxious mother that he wasn't doing enough work and would have to answer to the trustees of the Allen Scholarship for misuse of their money), Ramsey returned to Cambridge in October 1924

[1] Ramsey's notes on the Axiom of Infinity are certainly later than this since they refer explicitly to the 'extra proposition' which Wittgenstein wrote in Ramsey's copy of the *Tractatus* during that visit.

[2] Letter in Moore papers, Cambridge University Library.

and immediately took up a teaching fellowship at King's College. It seems very unlikely that he would have had very much time for research during his first term in this post. Fellows of King's were worked quite hard in those days and he would probably have had 12 hours a week of undergraduate supervisions to give in his first term.

After his first term as a teaching fellow was over, though, he would have had a little time to write up the work he had done before and during his stay in Vienna in the form of an essay. We know from a letter he wrote to Lettice[3] (with whom he had at this point only just started a relationship) that he sent the essay to be typed on 31st December 1924. This was just in time for it to be submitted as an entry for the Smith's prize (a competition for dissertations by beginning graduate students in the Cambridge Mathematics Faculty) at the beginning of the Lent Term (i.e. mid-January) 1925.

The essay did not win the Smith's prize, which went to a contemporary of Ramsey at St John's College called Gerald Room.[4] The following summer, however, Ramsey decided to submit the essay for publication. The impetus for this was the reforms imposed on the university by the Oxford and Cambridge Act of 1925. Until then the vast majority of the teaching staff at Cambridge did not hold office in the university itself but received their earnings by means of stipends from the colleges at which they held their fellowships, which they supplemented by charging a guinea for each student who attended one of their lecture courses. The new act of parliament led to the establishment of a reformed employment structure (which has survived in its essentials to the present day) in which the normal post for most of the university's academic staff was to be the office of University Lecturer. Ramsey intended to apply for one of these newly created posts, but if he was to do so he needed some more publications. One product of this sudden need to publish was his paper on 'Universals', which he wrote (apparently in something of a rush) and submitted to *Mind* in the summer of 1925. Another was that Ramsey decided to try to publish his prize essay. In those days before double-blind refereeing, however, he was worried that a journal which had not heard of him might reject it, so on 24th July 1925 he wrote to Russell[5] asking for a letter of support to be included with the paper so as to ensure that the journal editor took it seriously.

Russell's reply has not survived, but Ramsey submitted his paper to the *Proceedings of the London Mathematical Society*, with or without Russell's testimonial: it must have been accepted in September or October of 1925, since we know that it was read at a meeting of the London Mathematical Society on 12th November 1925. As I have been unable to trace a copy of the prize essay in the form it was originally submitted as an entry for the Smith's prize, it is a matter of conjecture how much Ramsey altered it between then and when it was published, but circumstantial evidence suggests that any changes were very minor. Certainly by the time he corrected the proofs of the published article, in late July 1926, he had

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[3] Letter in Modern Research Archive, King's College, Cambridge.

[4] Room's essay was called 'Varieties generated by collinear stars in higher space'. It would of course make a good story if Room had sunk without trace, but in fact he had a distinguished career as a geometer, was a founding Fellow of the Australian Academy, and had the mathematics library at the University of Sydney named in his honour.

[5] Letter in Russell Archives, McMaster University.

thought of ever so many ways in which if I hadn't been damned slack I'd have made it better. That always happens at least also with my Universals paper; I never write anything except in a hurry because it is pressing and then am too slack and self-satisfied to improve it afterwards at my leisure.[6]

And there are various places in which the published article betrays its origins as a prize dissertation. It starts with a table of contents, for instance, which is a little unusual in a paper of this length (47 pages).

What is at any rate clear is that the overall content of the published paper did not advance significantly beyond the prize essay. We know this because in the Lent Term 1925, immediately following his submission of the essay, Ramsey gave for the first time a lecture course on 'Foundations of mathematics' and lecture notes taken by a student at these lectures survive.[7] Ramsey included in this course a summary of his own work and included in this summary all the key ideas of the published paper.

Where does all this leave the dating of our transcendental argument? As we have seen, Ramsey adopted the idea of propositional functions in extension some time between February and September 1924, most likely during his stay in Vienna. The transcendental argument must date from before this adoption. On the other hand, a set of notes on 'Identity', with which his notes on 'The infinite' are closely related, make use of Wittgenstein's translation of 'Something other than *a* is *f*', which Ramsey did not receive until about the beginning of December 1923. All of this suggests that the argument dates from some time between January and September 1924. But the influence of Kant is also evident in the notes, not only in the form of the argument itself, but in a contrast Ramsey draws between intuitive and discursive mathematics, and we know from his diary that Ramsey was reading Kant early in 1924. This makes it rather plausible that 'The infinite' may well be what Ramsey is referring to in his diary entry for 28th January 1924: 'Wrote after tea some notes on formal logic (abstraction, identity, axiom of infinity).'[8]

IV

The conclusion Ramsey was trying to substantiate, that mathematics consists of tautologies, is one to which Wittgenstein was fundamentally opposed. It was therefore important to him to object to some part of Ramsey's theory. However, what he objected to was not so much Ramsey's failure to provide a good argument for the Axiom of Infinity as the other part of his account, the theory of propositional functions in extension. Some of his objections to this theory are contained in a letter he dictated to send to Ramsey in July 1927 (McGuinness 1995, pp. 216-8) – just about the first evidence we have of Wittgenstein doing serious philosophy after his long sabbatical in Lower Austria. But Wittgenstein did not rest there: he returned to the issue in *Philosophical Remarks* (§120) and *Philosophical Grammar* (pt. II, ch. III, §16), struggling to find a formulation which expressed his objection clearly.

The issue of whether Ramsey's notion of a propositional function in extension is coherent lies at the centre of deep difficulties in modern set theory on which I cannot

[6] Letter to Lettice Ramsey in the Modern Research Archive, King's College, Cambridge.

[7] L. H. Thomas papers, Special Collections Department, North Carolina State University Library.

[8] The diary is in the Modern Research Archive, King's College, Cambridge.

arbitrate here. But there is at the very least reason to think that Wittgenstein may have been right (see Sullivan 1995). In which case it behoves us to return to Ramsey's argument and ask, if we suppose that its first premiss is reinstated, whether its second and third premisses are likewise in good order.

Ramsey's second premiss, let us recall, was that if it is meaningful to say that there are infinitely many empirical entities (e.g. physical atoms or protons) then there must be some type of which it is meaningful to say that there are infinitely many objects of that type. Or, more briefly, if $q_{\aleph_0}$ is meaningful, $p_{\aleph_0}$ is meaningful. Is this true?

There have of course been at various times scientists who have been physical atomists – who have supposed, that is to say, that the physical world is made up of irreducible entities of certain kinds. What these kinds have been has varied. At one time the irreducible entities were thought to be atoms (hence the name). During most of the 20th century schoolchildren were taught that the world is made up of electrons, protons and neutrons. Nowadays the fundamental particles are much more exotic. But at any of these stages in scientific development it would have been possible to represent the world in such a way that the physically irreducible entities are also logical atoms, i.e. Tractarian objects. For anyone who did represent the world in this way the required link between $q_{\aleph_0}$ and $p_{\aleph_0}$ would of course be trivial.

But the physical atomist is not *forced* to take this step. For in order to qualify as a physical atomist all one is required to believe is that the world is *in fact* made up of such and such fundamental particles, not that it *must* be. And a physical atomist who thought that the particles which are in fact fundamental might not have been would on Wittgenstein's account have to represent the world in such a way that these particles are not Tractarian objects.

So the fundamental particles of physics might not be Tractarian objects. However, Ramsey's argument does not require them to be. Nor does it even require there to be any fundamental particles. For suppose that there are only finitely many Tractarian objects and finitely many elementary propositions. Since every proposition is according to the *Tractatus* expressible as a truth-function of elementary propositions, it follows that if we count propositions according to sense there are only finitely many different propositions. But Ramsey's point is that I can conceive of the possibility of saying 'Here is a particle' of infinitely many 'here's, i.e. of infinitely many possible utterances with different senses. This contradicts the supposition that there are only finitely many objects.

But if this re-establishes Ramsey's second premiss, it shifts the focus all the more acutely to his third, namely that $q_{\aleph_0}$ is indeed meaningful. Is this true?

One worry we might have about this premiss concerns the Aristotelian distinction between the actual and the potential infinite: perhaps what we mean when we say that there are infinitely many physical things is only that for every finite number n there *could* be n physical things, not that there actually are infinitely many. But in fact this is enough for our purposes, since if the proposition q_n that there are n physical things is meaningful, then the logical product of the q_n for all finite n is also meaningful, and we can let that be our $q_{\aleph_0}$.

The matter is rather delicate, however. We cannot be too liberal in our allowance of situations whose possibility we can represent to ourselves, for otherwise we fall into the opposite difficulty that the argument may prove too much. More precisely, if κ is any infinite cardinal, it seems as though we can meaningfully (although perhaps falsely) say that there are κ many physical things in the world. If so, then an argument parallel to Ramsey's allows us to deduce transcendentally the proposition p_κ that there are κ Tractarian objects of some type. But this evidently very dangerous: in the simple theory of types which Ramsey was recommending it would in fact be inconsistent.

If we are to block this inference to the existence of ever more objects, we need there to be a limit to the number of things our representation of the world allows for. What we presumably have to do here is to observe a distinction between what our current representation of the world allows as possible and what other representations are possible.

Ramsey's argument thus has to be that our current representation of the physical world *already* allows for the possibility that it is infinite. We could perhaps go further and say that the modern physicist standardly represents the world as having $2^{\aleph_0}$ points. But we do not already represent the world as having κ possible locations in it for larger cardinals κ. To allow for such a possibility – to make q_κ meaningful – would involve an enlargement of the number of objects in the world (or, to put it in more modern jargon, a change in our conceptual scheme).

In order to keep this distinction, therefore, Ramsey has to maintain that (1) we understand at the moment what q_n means for every finite n, but (2) there are larger cardinals κ for which we do not at the moment understand what q_κ means. Towards the end of his manuscript on 'The infinite' Ramsey puts the points as follows:

> The whole trouble with the infinite is that we cannot get at it directly. 3 we can get at at once by a tripartite symbol, that is by intuition (Anschauung) but we cannot make infinitely complex symbols. (But we may have such in spatial images, yet I think never do for they are not infinitely differentiated, but perhaps the image of motion may be really useful.) ... Hence at a certain point probably I think where we deal with infinities mathematics must cease being intuitive and become discursive. We must describe infinite cardinals in the manner of Cantor instead of seeing them as we can the finite integers. (Ramsey 1991, p.181)

But when it is put like this, Ramsey's original claim begins to seem dubious. Of course we can utter q_n and think that we understand it. But we can just as well utter, and seem to understand, q_κ for some large cardinal κ. The question in both cases is whether we really understand it. Suppose that there are in fact only N physical things in the world. Can I really in that case understand q_{N+1}? The question, remember, is not whether I can envisage adopting a new scheme within which N+1 things are represented, but whether my *existing* scheme of representation already allows one to say (falsely, as we are supposing) that there are N+1. Or, to put the matter in Ramsey's Kantian manner, how can I be sure that my understanding of large finite numbers is not *already* discursive rather than intuitive?

At this point, though, we have arrived at one way of expressing the central concern which occupied Wittgenstein when he returned to Cambridge in 1929 to work once again with Ramsey on the foundations of mathematics, a collaboration that was cut short at the end of that year by Ramsey's illness and death. It is rather striking that this concern, which occupied Ramsey so much at the end of his life, was already present in his notes on 'The infinite', since, as we have seen, they are probably among the earliest things on the foundations of mathematics that he wrote.

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