1A SRP WORKSHEET 2 SECTION A SOLUTIONS

NAME: CLASS: TUTOR:

5 Excellent
4 Good
3 Satisfactory
2 Weak
1 Very poor

Reading
Steinhart, *More Precisely* Ch. 5.
Kyburg, *Probability and Inductive Logic*, Ch. 2

SECTION A

1. You make two tosses of a coin that lands heads half the time. You want to know whether it will land heads this time.

   (a) Write down the event space for the outcome of the first toss

   V = **{H, T}**

   (b) Write down the field for the first toss

   $\wp$ (V) = **{∅, {H}, {T}, {H, T}}**

   (c) Write down the event space for the outcomes of both tosses

   V^2 = **{(H, H), (H, T), (T, H), (T, T)}**

   (d) What is the probability that it lands heads at least once?

   Pr ({(H, H), (H, T), (T, H)}) = ¾

   (e) What is the probability that it lands heads twice?

   Pr ({(H, H)}) = ¼

   (f) What is the probability that it lands heads at least once given that it lands tails at least once?

   Pr (Heads at least once) = ¾ by part (d)
   Pr (Tails at least once) = ¾ by the same reasoning
   Pr (Heads at least once │ Tails at least once)
   = Pr (Heads at least once ∩ Tails at least once) / Pr (Tails at least once)
   = Pr ({(H, T), (T, H)}) / (3/4)
   = (1/2) / (3/4) = **2/3**

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2. Two cards are drawn at random and without replacement from a standard pack of 52 cards. What is the probability that:

(a) Both are aces

The event space has 52×51 elements (because there are no replacements). So each possible draw has a probability of 1/(51 × 52).

How many of these draws involve two aces? They are as follows:

(AS, AH)
(AS, AD)
(AS, AC)
(AH, AS)
(AH, AD)
(AH, AC)
(AD, AS)
(AD, AH)
(AD, AC)
(AC, AS)
(AC, AH)
(AC, AD)

There are 12 of these, so the answer is **12 / (51 × 52) (**you can leave it like that).

(b) One is an ace

There are:

4 × 48 events in which the first is an ace and the second is not
48 × 4 events in which the first is not an ace and the second is
12 events in which both are aces

So Pr (One ace) = (48 × 4 + 4 × 48 + 12) / (51 × 52)
= **396/(51 × 52)**

(c) Both are aces given that one is an ace.

We want Pr (Both are aces | One is an ace). This is

Pr (Both are aces ∩ one is an ace) / Pr (One is an ace)
= Pr (Both are aces) / Pr (One is an ace).

Using (a) and (b) we know that Pr (Both are aces) = 12/(51 × 52), and we know that Pr (One is an ace) = 396/(51 × 52). Dividing the first quantity by the second we see that the answer is **12/396 = 1/33**.

(d) Both are aces given that one is the ace of spades.

Pr (Both are aces | One is the AS) =
Pr (Both are aces ∩ One is the AS) / Pr (One is the AS)

First work out Pr (Both are aces ∩ One is the AS). Here are the events on which that holds:

(AS, AH)
(AS, AD)
(AS, AC)
(AH, AS)
(AD, AS)
(AC, AS)

6 such events, so Pr (Both are aces ∩ One is the AS) = 6 / (51 × 52).

Now work out Pr (One is the AS). There are 51 events in which the first is the AS and 51 events in which the second is the AS.

So Pr (One is the AS) = (51 + 51) / (51 × 52) = 102 / (51 × 52).

So Pr (Both are aces | One is the AS) = **6 / 102 = 1 / 17**.

(e) The second is an ace given that the first is an ace

We need to find:
Pr (First is an ace | Second is an ace)
= Pr (First is an ace ∩ Second is an ace) / Pr (First is an ace)

First calculate Pr (First is an ace). How many events are there in which that happens? Well, there are 51 cases in which the first is the AS, 51 in which it is the AH, 51 in which it is the AD and 51 in which it is the AC. So there are 4×51 = 204 such events. So Pr (First is an ace) = 204 / (51 × 52)

Now calculate Pr (First is an ace ∩ Second is an ace)
= Pr (Both are aces)

We know from part (a) that this is 12 / (51 × 52).

So:
Pr (First is an ace | Second is an ace)
= Pr (First is an ace ∩ Second is an ace) / Pr (First is an ace)
= (12/(51 × 52)) / (204/(51 × 52) = 12/204 = **1/17**

(f) The first is an ace given that the second is an ace

Same as (b) except ‘first’ and ‘second’ are permuted: **so 1/17 again**.