What is a predicate?

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According to Frege, the sentence 'Socrates is mortal' can be analysed into the proper name 'Socrates' combined with the one-place predicate 'ξ is mortal'. It is uncontroversial that the two items are governed by different grammatical rules of combination. But Frege also introduced semantic distinctions between them that have been contended ever since. He made them refer to different types of thing. The proper name refers to an *object*, while the predicate refers to a *concept*, which he proceeded to identify with a function from objects to truth-values, e.g. from Socrates to truth. His distinction between objects and functions is exclusive: nothing can be both. He marked the difference by saying that an object is complete or saturated, whereas a function is incomplete or unsaturated. Proper names and predicates also differ in the way in which they refer to their referents. Frege uses 'refers to' as an umbrella term covering different relations, since his principle for individuating them dictates that the reference relation holding between a proper name and an object is of a different type from the relation holding between a predicate and a concept. He made further semantic distinctions between proper names and predicates at the intermediate level of *sense*, but discussion of them will not be necessary, except in §2.4.

These syntactic and semantic contrasts between proper names and predicates are relational in character. My question is whether proper names and predicates also differ

in their intrinsic nature. Are they different sorts of thing, as different perhaps as the corresponding objects and concepts?

Frege answered that proper names and predicates 'differ essentially' (1904b, p. 161). Just as nothing can be both an object and a function, so nothing can be both a proper name and a predicate. He characterised the difference between them by redeploying the adjectives that he used to contrast their worldly referents. Proper names, like objects, are said to be complete and saturated; predicates, like concepts, are not. There is little agreement about what Frege meant by this or whether he was right. The dispute focuses on predicates, since it is generally accepted that proper names can straightforwardly be construed as expressions. In Part 2 I shall describe and evaluate Frege's account of predicates, taking issue with other exegetes. But first I need to map out various candidates for predicates and investigate the apparent competition between them.

Part 1. Predicates

1.1 Four candidates

Plain expressions   I start with the simplest case, an atomic sentence '*Fa*' of the predicate calculus. Generations of logic students have learned to identify the expression '*F*' as the predicate occurring in '*Fa*'. Like the sentence, the expression is an abstract type. It may have any number of concrete tokens, of different forms and in different substances. And the type itself may re-occur in other sentences, or within a single compound sentence. But its grammar dictates that it occurs only once in an *atomic* sentence—at the front—and is followed by just one term. The grammatical

rule governing the predicate '*F*' invokes a general method of constructing sentences, namely *predication* of a one-place predicate (we shall come to predicates of higher degrees when the need arises). This method of construction may be identified with the *linguistic* function that maps two arguments—a one-place predicate φ and a term α—to a single value—the sentence φα.

Some authors characterise predicates in just these constructive terms: 'Expressions that yield sentences when thus attached to singular terms are called *predicates*' (Massey 1970, p. 226). One might equally well come at predicates by decomposition or subtraction: '*F*' is what is left when '*a*' is subtracted from '*Fa*'. A third account would emphasise the sharing of a single predicate by related sentences: '*F*' is the common element among '*Fa*' and its substitutional variants '*Fb*', '*Fc*' etc. Nothing hinges on the differences between these descriptions. They describe the same thing: the expression '*F*'.

I now introduce three further candidates, each intimately associated with the expression '*F*', and each leading naturally to the next. Each has some right to be called a *pattern*, but to avoid conflation and confusion, I give them different names.

Schemata   The sentences '*Fa*', '*Fb*', '*Fc*' etc share a pattern '*Fx*'. In one sense of pattern, the common pattern is the very expression '*Fx*' (see e.g. Kirwan 1978, p. 3). The '*x*' in '*Fx*' functions as a schematic letter, and '*Fx*' is itself sometimes called a schema, which is the label I shall use; I sometimes call '*F*' a *plain* expression to distinguish it from the schema '*Fx*'. Accordingly, when one says that '*Fa*' etc share the pattern '*Fx*', one means that they are each the result of substituting a term for '*x*' in '*Fx*'.

Linguistic functions   The substitution procedure applied to the schema '*Fx*' yields a corresponding linguistic function: *the result of substituting the term … for 'x' in*

'*Fx*'. Call this function *f* for short, then the sentence '*Fa*' is the value of *f* for the term '*a*' as argument, while the variants of '*Fa*' are values of the same function for different arguments.

According to a second sense of 'pattern', the pattern common to '*Fa*' and its variants is not the schema '*Fx*' but the corresponding linguistic function *f*. So when we say that '*Fa*' and its variants all share or illustrate a common pattern, we now mean that they are all values of the same function for different arguments (see e.g. Geach 1975, pp. 142–3).

Instead of using an '*x*' in a schema, others might use a '*y*', or dots or lines or circled numerals or Greek consonants ('*F...*', '*F___*', '*F①*', '*Fξ*'). The choice is plainly arbitrary, but it prompts a search for a sense of pattern according to which these trivially different schemata all depict the same pattern. Our second sense of pattern gives us what we want, since the same linguistic function may be described in terms of substitution of a term for e.g. the letter '*x*' in '*Fx*', or the dots '*...*' in '*F...*'. Of course, it may also be described without making any reference to a schema, e.g. *the result of attaching the term ... to the expression '*F*'*.

The linguistic function *f* is related to but distinct from the linguistic function that I earlier identified with the predication construction. The latter takes *two* arguments, e.g. it maps '*F*' and '*a*' to '*Fa*', whereas *f* takes just *one*, e.g. it maps '*a*' to '*Fa*'. In effect, the expression '*F*' has been absorbed into the function *f*.

Properties of sentences   The sentence '*Fa*' is the value of the function *f* for the argument '*a*'. Hence '*Fa*' has the property *being the value of f for some term as argument*, a property which it shares with its variants '*Fb*', '*Fc*' etc. This common property may be called a pattern in yet another sense of the word (see e.g. Dummett 1973, pp. 31, 246, 250, 412, who also speaks of 'features' of sentences).

I have derived the common property from the corresponding linguistic function. But it too can be described more directly, e.g. as the property *consisting of 'F' followed by a term*. As well as depicting the function *f*, the schema '*Fx*' may be understood in a different way as depicting this property of sentences.

As I have explained, the pattern common to '*Fa*' and its variants may be regarded as either a schema, or a linguistic function, or a property of sentences. Indeed, the relevant *predicate* may be regarded as any of these things. Including the plain expression '*F*' with which we started, we now have four candidates. So: what is a predicate?

Different authors have given different answers. One has even given different answers at different times. Geach started by defining predicates as expressions: 'A *predicate* is an expression that gives us an assertion about something if we attach it to another expression that stands for what we are making the assertion about' (1950, p.461). Within three years, however, he had come to deny that a predicate is an actual expression: 'I should rather regard it as a common property of sentences' (1953, p. 224). Finally he championed linguistic functions:

> in 'Raleigh smokes', let us say, the two terms have totally different modes of significance. 'Raleigh' signifies just by being a man's name. We cannot sensibly ask what 'smokes' means; what is significant is not the bare word 'smokes' but a certain pattern—name followed by 'smokes'. And speaking of a common pattern in 'Raleigh smokes', 'Churchill smokes', etc., is just another way of saying what Frege would have expressed by saying we had values of a common function for a series of different arguments—the names 'Raleigh', 'Churchill', etc. (1975, pp. 141–2)

1.2 Anything goes

Whereas other authors argue for different candidates, I propose that there is nothing to choose between them. Anything goes: each is equally serviceable. The rich multiplicity of candidates (there will be more) is not an embarrassment. The choice between them can be made arbitrarily, or when the context allows, it can be left unmade.

I therefore need to rebut arguments that seek to show that some candidates are not fit for purpose. In the literature, the objections are targeted against plain expressions and against schemata (Geach himself subsequently admitted that the difference between linguistic functions and properties of sentences 'seems to matter little' (1976, p. 61)). Before rebutting them, it is important to emphasise that for any choice of candidate, we can give an appropriate sense in which a predicate *occurs* in a sentence, and so an appropriate sense in which a predicate can be *shared* by different sentences. When a predicate is construed as a plain expression like the '*F*' in '*Fa*' and '*Fb*', it occurs in them by simply being a part of them, albeit in a sense of 'part' that is appropriate for abstract types rather than concrete tokens. If another candidate for predicates is selected, an obvious compensating change must be made to the sense of 'occurs'. The schema '*Fx*' occurs in '*Fa*' means that '*Fa*' is the result of substituting a term for '*x*' in '*Fx*'. The associated linguistic function *f* occurs in '*Fa*' means that '*Fa*' is the value of *f* for some term as argument. And the corresponding property of sentences occurs in '*Fa*' simply means that it has the property. These accounts of occurrence are tailor-made for atomic sentences. More complex contexts demand more complex accounts of occurrence. But since all four candidates are so closely linked, what works for one can easily be adapted for the others.

1.2a Objecting to schemata

In the last paragraphs there is a certain linguistic awkwardness, of some philosophical interest. It is natural to speak of … the two-place predicate 'ξ killed ζ'; but here, as Frege would put it, by a kind of linguistic necessity we cannot quite say what we are trying to say … if we speak of the predicate 'ξ killed ζ' as figuring in 'John killed Mary' or 'Mary killed John', then again what we quote does *not* figure in these sentences. The actual expression 'ξ killed ζ' is neither a function nor a predicate: it serves however to identify a two-place predicate shared by many sentences, and this is the same thing as identifying a function yielding such sentences as 'John killed Mary' and 'Mary killed John' as its values when proper names are supplied as its arguments. (Geach 1975, pp. 148–9)

Geach here thinks that the phrase 'the two-place predicate "ξ killed ζ"' can only be understood as picking out a schema. Since he regards predicates as linguistic functions, 'we cannot quite say what we are trying to say'. But he himself allows that schemata depict (or, as he puts it, 'identify') linguistic functions. So in fact we could use another mode of expression to say what we are trying to say, namely 'the two-place predicate depicted by "ξ killed ζ"'. There is no need to do so, however. His account of quotation is too restrictive. The phrase 'the two-place predicate "ξ killed ζ"' does not pick out a schema as a matter of 'linguistic necessity', since we can understand the quotation marks to be directing us towards any of the candidates for predicates, without the detour via depiction by an intermediary schema. Some famous philosophers' pronouncements notwithstanding, material enclosed in quotation marks

may stand for a wide variety of things. Common examples are: expression tokens, whether written or spoken, meaningless or meaningful; expression types for all the different ways of typing them; meanings. As the Kneales quipped: 'Quotation marks were made for man, not man for quotation marks' (1962, p. 514). Logicians' understandably baulk at such ambiguity, and invent different styles of quotation marks in order to resolve it. But there is a more relaxed way to disambiguate in ordinary use, namely the addition of an explanatory prefix, as in 'the printed token "loves"', 'the phonological type "loves"', 'the lexeme "loves"'.

To take our current topic, then, suppose that predicates are construed as linguistic functions. By using the prefix in 'the linguistic function "ξ killed ζ"' we can pick out the relevant function directly. Once it is settled that linguistic functions are the chosen candidates, the same goes for 'the two-place predicate "ξ killed ζ"'.

This point about quotation aside, why does Geach suppose that we cannot really mean to mention the schema when we use the phrase 'the two-place predicate "ξ killed ζ"', i.e. why cannot predicates be schemata? He claims that the schema 'ξ killed ζ' does not occur (or, as he says, 'figure') in 'John killed Mary', evidently on the ground that the schema contains Greek letters while the sentence does not. He has failed to understood 'occur' in the sense appropriate to the candidate. The schema 'ξ killed ζ' does indeed occur in 'John killed Mary' in the relevant sense, since the sentence results from substituting terms for the schematic letters. Geach cannot object to this reinterpretation of 'occur' to suit the candidate, for he himself must say that a linguistic function—his favoured candidate—occurs in the sentence in a sense of 'occur' quite different from that in which the plain expression 'killed' occurs.

1.3 Objecting to plain expressions

I now turn to a pair of arguments that aim to show that some or all predicates cannot be plain expressions. The first is Dummett's (1973, pp. 27–33). He draws a distinction between two kinds of predicate according as they play different explanatory roles. *Simple* predicates are needed to explain the syntactic and semantic structure of atomic sentences. *Complex* predicates, on the other hand, are introduced to explain the structure of sentences that feature variable-binding devices such as quantifier phrases, and to explain and to represent schematically the validity of arguments featuring such sentences. To use Dummett's own examples, 'Brutus killed Caesar' contains the simple predicate 'ξ killed ζ', which is used in accounting for the construction and understanding of the sentence. But we might also need to represent the sentence as containing the complex predicate 'ξ killed Caesar' when explaining and representing the validity of an argument in which the original sentence figures alongside the quantified 'anyone who killed Caesar is an honourable man'.

Dummett claims that this difference in role is matched by a difference in nature: simple predicates are not the same sort of thing as complex ones. He asserts that simple predicates are plain expressions. The Greek letters in 'ξ killed ζ' are not elements of the simple predicate represented, but are merely used to indicate the location, nature and number of its arguments (this is another example of how material enclosed in quotation marks may be used to pick out different candidates for predicates in different contexts). Dummett quite reasonably counts even a discontinuous string of words as a plain expression, e.g. the simple predicate 'ξ took ζ to task' consists solely of the two discontinuous parts 'took' and 'to task'.

In contrast to simple ones, complex predicates are not expressions at all, but 'features' or 'patterns' of sentences, by which Dummett means properties of them. In order to establish this difference in nature, he focuses on just one, quite special kind of complex predicate, namely those that have argument-places that are, as Frege (1893, §4) says, 'related', i.e. places that must be occupied by occurrences of the same term. Related argument-places are indicated by repeating Greek letters, as in 'ξ killed ξ'. According to Dummett,

> There is no part in common to the sentences 'Brutus killed Brutus' and 'Cassius killed Cassius' which is not also part of the sentence 'Brutus killed Caesar': yet the predicate 'ξ killed ξ' is said to occur in the first two and not in the third. Such a complex predicate is, rather, to be regarded as a *feature* in common to the two sentences …it does not consist merely of some sequence of words or symbols … the complex predicate is thus not really an expression—a bit of language—in its own right (1973, p.31)

It is plain that Dummett's argument is limited in scope, since it only applies to predicates with related places. For all that he has said, a complex predicate without related places, e.g. his 'ξ killed Caesar', can be construed as a plain expression. Moreover, even if the argument works for predicates with related places, it can only show what they are *not*. It cannot establish Dummett's positive conclusion that complex predicates are properties of sentences. Why not schemata or linguistic functions? The same may be said against Geach's use of an identical argument, first to argue that predicates are properties of sentences (1953, p. 224), and later to argue that they are linguistic functions (see e.g. his 1975, pp. 139–40).

In any case, the argument does not even succeed on these limited and negative terms. Dummett tells us that an expression is 'a sequence of phonemes or of printed letters' or a word or string of words 'which can quite straightforwardly be written down' (1973, p. 32). Yet even simple predicates cannot be expressions in this attenuated sense, since a predicate needs to be distinguished from its homophones and homographs, which may not obey the same rules of combination. In other words, if we use Dummett's notion of an expression, a simple predicate cannot be an expression *simpliciter*, but only an expression coupled with a grammatical rule. He himself notes that his simple predicate 'killed' is governed by the grammatical rule that it goes with two terms, *the same or different*, one on each side. But then he must allow that the same expression 'killed' may be governed by the different grammatical rule that it goes with the *same* term on either side. Although the expression remains constant, the predicate is different, since the expression is coupled with a different grammatical rule. Of course, inspecting the plain 'killed' does not by itself reveal the intended predicate's grammar. But it can be easily described, or else indicated by letters within quotation marks: 'ξ killed ζ' vs 'ξ killed ξ'.

It follows that simple and complex predicates, even those with related places, swing together ontologically. In particular, both can be regarded as expressions, in Dummett's sense, coupled with grammatical rules. Nothing changes if expressions are individuated more finely, by building grammatical rules into their identity conditions. It would still be true that both kinds of predicate may be construed as expressions. This not to deny that simple and complex predicates play different explanatory roles, but it is to deny Dummett's thesis that the distinct roles are reflected in distinct natures.

Noonan tackles both predicates and functors in one go, and gives an altogether

different argument against regarding them—all of them—as plain expressions. He favours regarding them as linguistic functions (his 'patterns'), and claims that we are inclined to regard them differently as expressions because

> in writing down the patterns exhibited by complex designations of numbers (like '2 + 3') or sentences (like 'Socrates is wise') we typically employ auxiliary expressions ('+', 'is wise') to construct the patterns thus exhibited. But we do not *always* do so (in mathematical symbolism the sign for the two-argument function *x raised to the power y* is 'x^y' and here there is no separable auxiliary expression which anyone could regard as the sign for the function), and we need *never* do so. (2001, p. 147)

It is true that '2^4' contains no expression standing for the function involved. It features only an *arrangement* of the terms '2' and '4'. But from the facts about this particular case it hardly follows that *no* functor, actual or possible, can be construed as an expression. When an 'auxiliary' expression does happen to be present, as in '2 raised to the power 4', it is a serviceable candidate for the relevant functor. Again, that there is a *possible* language in which no functor is an expression (Noonan's 'we need *never* do so') is compatible with the fact that in *our* language expressions comprise one kind of candidate for many functors. As for functors, so for predicates.

   Noonan works with a crude dichotomy between two candidates: plain expressions vs linguistic functions. But contrary to his intentions, his example actually serves to introduce yet another candidate—a fifth—for predicates and functors, which I omitted from the initial list in §1.1. In '2^4' the relevant functor may be identified with the arrangement of the two terms, i.e. a particular relation holding between them. Turning

to predicates, one candidate for a predicate in 'Brutus killed Caesar' is the relation that holds between two terms, the same or different, when one is to the left of the expression 'killed' and the other to the right. This idea will be familiar to readers of the *Tractatus* (Wittgenstein 1922, 3.1432). The corresponding candidate for a one-place predicate is not a relation between terms but a property of a single term, which is distinct from but related to the property of *sentences* discussed in §1.1.

Part 2. Frege

2.1 Frege's own candidate: expressions with empty places

I have now assembled almost all of the materials required to understand and evaluate Frege's own account of predicates. I need to add two points about his logic which I have so far glossed over. First, he uses 'proper name' in an idiosyncratic, extended sense as including not only names such as 'Socrates' but also complex singular terms such as definite descriptions, and functional value terms obtained by applying a functor to its arguments, like '2+3' (see e.g. his 1892a, p. 158). The second point concerns the kinds of linguistic item that take arguments and produce values. As with the case of 'occur' in §1.2, 'take' and 'produce' can be understood neutrally, or they can be given a specific sense appropriate to a particular candidate. It is now usual to distinguish such items according as their arguments are singular terms or sentences and their values are singular terms or sentences. Predicates take singular terms and produce sentences, sentential connectives take sentences and produce sentences, while 'functor' is often reserved (as in §1.3) for items that take singular terms and produce

singular terms. Frege, however, lumps them all together as 'function-names'. The
reason is a second idiosyncrasy, namely his misconceived assimilation of sentences to
singular terms. They too count as 'proper names' (see e.g. 1892a, p. 163). He does
indeed speak more specifically of concept-words and relation-signs, but these do not
correspond to the modern notions of one- and two-place predicates, since they can
take sentences as arguments, e.g. '$(2 + 3 = 5) = (2 = 2)$' is well-formed. (He
distinguished function-names according to the 'level' of the functions for which they
stand, e.g. higher-level functions that take other functions as arguments. But I shall
have nothing to say about them, since they raise no new points of principle.)

In the passages I shall be quoting, therefore, Frege is out to characterise the nature
of function-names in general, rather than predicates in particular. But it should already
be clear from the discussion of Noonan in §1.3 that the multiplicity of candidates for
predicates is easily replicated for connectives and functors.

Frege devotes §1 of *Grundgesetze* to explaining function-names and the functions
they name. He first warns against confusing functions with expressions:

> If we are asked to state the original meaning of the word 'function' as used in
> mathematics, it is easy to fall into calling a function of *x* an expression, formed
> from '*x*' and particular numbers by use of the notation for sum, product,
> power, difference, and so on. This is incorrect, because a function is here
> represented as an *expression*, as a concatenation of signs, not as what is
> designated thereby. (1893, §1)

He proceeds to argue that even a *function-name* cannot be regarded as an expression
featuring the letter '*x*' such as '$2+3x^2$', where we would call '*x*' a free variable. For

this kind of expression 'indeterminately indicates' a value of the relevant function, and therefore does not stand for the function itself. He does not conclude that function-names are not expressions at all, only that they are expressions of another, special kind: 'the expression for a *function* is *in need of completion, unsaturated*' (§1). What does this mean?

Frege repeatedly says that function-names are expressions with empty places. He means what he says; he never offers anything that would defeat a literal reading. I give four choice quotes; there are plenty more.

> the expression for a function must always show one or more places that are intended to be filled up with the sign of the argument (1891, p. 141)

> The name of a function is accompanied by empty places (at least one) where the argument is to go; this is usually indicated by the letter '*x*' which fills the empty places in question. But the argument is not to be counted as belonging to the function, and so the letter '*x*' is not to be counted as belonging to the name of the function either. Consequently one can always speak of the name of a function as having empty places, since what fills them does not, strictly speaking, belong to them. (1892–5, p. 119)

> every function sign must always carry with it one or more places which are to be taken by argument signs; and these argument places—not the argument signs themselves—are a necessary component part of the function sign. (1896, p. 116)

function names must differ essentially from proper names, the difference being that they carry with them at least one empty place—an argument place. And these argument places must always be preserved in a function name and be recognizable as such; otherwise the function name becomes a meaningless proper name. (1904b, p. 161)

Empty places are hard to see, especially when they come at either end. Frege uses different devices to make empty places visible. One is a pair of brackets serving as an empty container: '( ) = ( )'. Another is the use of Greek letters: '$\xi=\zeta$', or the extreme case 'ξ', where the letter indicates the bare empty place that is his function-name for the identity function mapping any object to itself (1893, §26).

He employs the Greek letters not just in the expressions he uses to mention functions, e.g. 'the function $\xi = \zeta$', but also in those he places between quotation marks to mention function-names, e.g. 'the function-name "$\xi = \zeta$" '. This fact might lead one to think that he regards function-names as schemata. Thus Potts: 'A Fregean incomplete expression or function name is the same as a schema; it always contains at least one schematic symbol' (1994, p. 71). But not so: Frege is clear that the Greek letters are not parts of his function-names.

where use is made of an expression like 'the function $\Phi(\xi)$', it is always to be observed that 'ξ' contributes to the designation of the function only so far as it renders recognisable the argument-places (1893, §1)

when we say 'the function $1 + \xi - \xi$', the letter 'ξ' is not part of the function-sign … the role of the letter 'ξ' is to enable us to recognize the places where

the supplementing proper name is to be put … This 'ξ' gives us a pointer for

how to use the function-name. (1914, pp. 239–40)

To repeat, the Greek letters do not here function schematically i.e. they are not parts

of the function-name that are *replaced* by argument-terms when the function-name

occurs in a larger context. They merely indicate empty places within the function-

name. The empty places are *filled* when the function-name occurs in a larger context.

(Note, too, that Frege's use of '$\xi=\zeta$' within quotation marks to mention an expression

with empty places is another case of the flexibility of quotation with respect to

reference.) So when he says that function-names are incomplete or unsaturated he

cannot be talking in a roundabout, metaphorical way about the role of schematic

letters within them, since they contain no such thing. Instead, his function-names are

incomplete in the *literal* sense that they carry with them argument-places conceived of

as 'empty places' or 'gaps'.

   To sum up Frege's view, the schema '$\xi=\zeta$' is not a function-name occurring in

'Hesperus = Phosphorus'. It contains too much. The plain expression '=', on the other

hand, contains too little: it omits empty places. A new kind of expression is needed:

viz. '   =   ' with its two empty places. Frege thus supplies us with a *sixth* candidate

for predicates.

2.2 Frege according to other commentators

Dummett's simple predicates are plain expressions. Although he acknowledges that

they may be assigned 'gaps' or 'slots', he takes this to be a metaphor for a

grammatical rule: 'the slot consists merely in the predicate's being subject to a certain

rule about how it can be put together with a term to form a sentence' (1973, pp. 32–3). (As noted in §1.3, a discontinuous predicate may be regarded as a plain expression that consists solely of its discontinuous parts, i.e. it has no gaps built into it.) Since Frege's talk of empty places is not metaphorical, his predicates cannot be Dummett's simple ones.

As for complex predicates, Dummett acknowledges that they 'form the prototype for Frege's general notion of an "incomplete" expression' (p.31). He notes that Frege says that they contain gaps, but again he wrongly makes such talk metaphorical, and now even more so. For he supposes that the incompleteness of an incomplete expression means that it is 'not really an expression—a bit of language—in its own right' (p. 31), but is instead to be regarded as a property of expressions. The same objection applies to Brandom's account (2000, pp. 131–2), since he identifies Frege's predicates with Dummett's complex ones.

A clue that something is wrong is that Dummett places all his emphasis on predicates with related argument-places. He believes that considering them makes Frege's notion of incompleteness 'immediately clear' (p. 31). Yet Frege himself does not single them out for special attention. On the contrary, he explains the idea of an incomplete expression by using predicates without related places (i.e. with either a single argument-place or several unrelated ones). For him, the incompleteness of an expression consists in its having at least one empty place. Incompleteness makes it a special kind of expression; it does not prevent it from being an expression. This holds even of predicates with related places. Frege needs only to indicate that their empty places are related, e.g. by using repeated occurrences of the same Greek letter as in '$\xi = \xi$' (contra Russell 1903, Appendix A, §482, who claims that Frege cannot indicate related places).

When Geach himself started to regard function-names as linguistic functions, he was hesitant about attributing the same idea to Frege:

> So far as I know, Frege never explicitly adopts the view that linguistic functions are what symbolize numerical (or other) functions; but it seems likely that he would have adopted it if it had been put to him. (1961, p. 144)

Fifteen years later, however, he threw off any pretence to exegetical caution, and applauded Frege's 'fundamental insight that a concept is represented not by an expression within a sentence but by a function from e.g. proper names to sentences' (1976, p. 440). Others who have claimed that Frege's function-names are linguistic functions include Hugly (1973), Noonan (2001, chs 2 & 4), Rumfitt (1994), and Sullivan (1992). Stenius swithers between linguistic functions and properties of sentences; compare footnotes 2 and 6 of his (1976). The same is true of the later Dummett. In *The Interpretation of Frege's Philosophy* he has Frege positing 'a congruence in logical type between the referents of expressions and the expressions themselves' (1981, p. 485). Proper names *stand for* objects and *are* objects. In contrast, 'what stands for something incomplete, a function, is itself incomplete', e.g. a predicate 'may be viewed as a common property of certain sentences, or a function whose values are those sentences' (p. 485). Here Dummett introduces linguistic functions as an alternative candidate for predicates, though he signally fails to decide between them and his earlier favourites, properties of sentences.

Identifying Frege's function-names with linguistic functions again wrongly imputes a metaphorical understanding of 'incomplete expression' and 'empty place'. It also strains credulity to suppose that he regarded function-names as functions, but felt no

need to make his view explicit or to warn against possible confusion. He is insistent about the need to distinguish a function-name from the function named. If he regarded function-names as themselves functions, he would surely have told the reader that they are *linguistic* functions, not to be confused with the functions they name. He did nothing of the sort.

Frege's division of things into objects and functions is exhaustive as well as exclusive. Since function-names are not themselves functions, they must be objects. On his view of a function-name as a kind of expression, this is just what one would expect. Indeed, we shall see in §2.3 that he explicitly relies upon the objecthood of function-names in order to circumvent his difficulty in talking about functions in the material mode (the paradox of the concept *horse*).

Pace Dummett, then, Frege did not posit a congruence in logical type between the referents of function-names and the function-names themselves. Function-names *stand for* functions but *are* objects. One must not be misled by his characterisation of function-names and functions as both incomplete. 'Incomplete' means different things when it qualifies different things. At the linguistic level, function-names are incomplete in the sense that they have empty places, as opposed to complete proper names. But when Frege moves from linguistic items to their worldly referents, he contrasts incomplete functions with complete objects. According to this second, worldly sense of 'incomplete', function-names are as complete as proper names, since they are both objects. Or, to come at the matter from the other direction, functions cannot be said to be incomplete in the linguistic sense, since they are not expressions.

It is true that Frege once contrived to see an analogy between the incompleteness of function-names and the incompleteness of functions, by modelling the second on the first (*not* vice versa). He called 'the meaning [i.e. reference] of a word part of the

meaning of the sentence, if the word itself is a part of the sentence' (1892a, p. 165),

and thus characterised functions, like function-names, as *incomplete parts* of a

complete whole. For example, the function-name 'ξ conquered Gaul' is an incomplete

part of the sentence 'Caesar conquered Gaul'. It is completed when the proper name

'Caesar' fills its empty place. By analogy, the function ξ *conquered Gaul* is an

incomplete part of each of its values; it is completed when it is applied to one of its

arguments ('the argument … goes together with the function to make up a complete

whole', 1891, p. 140)).

This attempt to endow functions themselves with something like empty places is

misconceived. It is utterly obscure how the values of the function ξ *conquered Gaul*,

namely truth and falsehood, can have any kind of part, let alone the function itself as a

part. Frege himself knew that he was on shaky ground in transferring the relation

between the parts and the whole of a sentence to their corresponding referents (1892a,

p. 165). But it was only late in his life that he presented a decisive objection. If a

function goes together with an argument to form a whole—its value for the given

argument—then the argument as well as the function must be a part of the value. Yet

'we cannot say that Sweden is a part of the capital of Sweden' (1919, p. 255).

At this point it is necessary to counter Geach's story about how Frege came to the

idea that predicates stand for functions. He tells us that

> Frege's first notion of a function was one that fitted only linguistic functions;
>
> but he later came to think that this view was insufficient—that functions
>
> belong to the subject-matter, not just the notation, of mathematics; his mind
>
> passed from linguistic functions, whose values and arguments are numerical
>
> expressions, to numerical functions, whose values and arguments are numbers;

so also it was natural that he should pass from the recognition of the linguistic functions that occur in predication to the view that there are functions in reality which these predicational functions represent. (1961, p. 151)

This is fiction from start to finish. In support of its opening assertion, Geach cites the following definition from *Begriffsschrift*:

Suppose that a simple or complex symbol occurs in one or more places in an expression …. If we imagine this symbol as replaceable by another (the same one each time) at one or more of its occurrences, then the part of an expression that shows itself invariant under such replacement is called the function; and the replaceable part, the argument of the function. (Frege 1879, §9; trans. Geach, p. 143)

But this definition actually contradicts his assertion that 'Frege's first notion of a function was one that fitted only linguistic functions'. It clearly applies 'function' to expressions, and as Geach himself insists, a linguistic function is not an expression, even when its arguments and values are. Thus the 'functions' of this definition are not linguistic functions.

The story ends by supposing that Frege actually regarded predicates as linguistic functions. This is hard to reconcile with Geach's admission a few pages earlier that Frege 'never explicitly adopts the view' (p. 144). Worse, Frege explicitly contradicts it when he talks literally of expressions with empty places.

In conceiving of functions as expressions, the youthful Frege was following the mathematical custom of his day. By 1891, however, he had clarified his ideas,

carefully distinguishing an expression for a function (later 'function-name') from the function itself, and convicting others of confusing the two: 'a mistake, admittedly, that is very often met with in mathematical works, even those of celebrated authors' (1891, p. 138. He was not given to self-criticism, but see the first note to §1 of his 1893, and also footnote 40 to Jourdain (1912), which Frege commented on in manuscript). He never again applies 'function' to function-names.

2.3 'On Concept and Object', footnote 8

One other text has been cited in favour of construing Frege's predicates (or, more generally, function-names) as linguistic functions, viz. footnote 8 to 'On Concept and Object' (1892b, p. 186). In this paper, Frege engages for the first time with the paradox of the concept *horse*. He takes it for granted that the expression 'the concept *horse*' is (i) a proper name in good standing, i.e. it (ii) expresses a full sense and (iii) refers. Since (iv) a proper name refers to a single object, if anything, and (v) nothing is both an object and a concept, it follows that 'the concept *horse*' refers to an object, not to a concept. He now faces 'a quite peculiar obstacle … a certain inappropriateness of linguistic expression' (pp. 193–4). Despite his best intention to mention a *concept* when using 'the concept *horse*', the expression itself refers to a proxy *object* that represents the concept. (Kerry, the critic to whom he is responding, had taken it to refer to something that is simultaneously an object and a concept, contrary to (v).) He also grants that 'ξ is not a concept' is (vi) a predicate, which is again in good standing, i.e. it (vii) expresses a full sense and (viii) refers. In particular, (ix) it refers to a concept that maps any object to truth, and therefore the sentence 'the concept *horse* is not a concept' is true. This is paradoxical because prior to meeting

Frege's semantic machinery, one would have intended the sentence to be false, and expected it to be so, by analogy with e.g. 'the volcano Vesuvius is not a volcano'. At this initial stage, Frege claims that (x) the paradox cannot be avoided: 'the obstacle is essential, and founded on the nature of our language' (p. 194), i.e. there is no other way of mentioning or saying what we intend.

Each of (i)–(x) has been rejected by some critic or other (not least Frege's later self) as a way out of the paradox. Solving it is not my present concern, however, since I only need to set enough of the scene in order to understand the relevant footnote. It reads:

> A similar thing happens when we say as regards the sentence 'this rose is red': The grammatical predicate 'is red' belongs to the subject 'this rose'. Here the words 'The grammatical predicate "is red" ' are not a grammatical predicate but a subject. By the very act of explicitly calling it a predicate, we deprive it of this property. (p. 186)

According to Hugly's reading of the footnote (1973, §IXA), Frege derives an analogous paradox concerning predicates from his view of them as functions. The proper name 'the grammatical predicate "is red" ' fails to refer to a predicate, since a proper name cannot refer to a function, and so the sentence 'the grammatical predicate "is red" is not a predicate' is true. Geach hints at the same reading. He too thinks that Frege foresaw that his difficulty with the concept *horse* arises 'on the linguistic level too … we see the futility of trying to escape Frege's difficulties by semantic ascent, by talking about words instead of the objects and concepts signified' (1975, p. 149).

But this reading of the footnote cannot be right. As noted in §2.2, all the evidence is against construing Frege's predicates as functions. And as we shall see, he explicitly argues that his difficulties *can* be circumvented by moving to the linguistic level. Worse, to read the footnote as presupposing that predicates are functions makes a nonsense of the main text of 'On Concept and Object', in which he explicitly construes them as expressions (the same goes for the contemporaneous 'Function and Concept', to which he points the reader in his concluding paragraph). He speaks of 'concept-words' interchangeably with 'predicates', and says e.g. that the *words* 'no other than Venus' stand for a concept. Or again, consider his account of why 'the concept *horse*' stands for an object, not a concept. If he had believed that it is linguistic functions which stand for concepts, he would have ruled out 'the concept *horse*' as standing for a concept simply on the ground that it is an expression, not a function. But he doesn't reason in this way. He denies that 'the concept *horse*' stands for a concept on the different ground that it is an expression *of the wrong kind*: 'this is in full accord with the criterion I gave—that the singular definite article always indicates an object' (p. 184).

Frege was soon to offer a solution to his difficulty in talking about concepts. He points out that 'the meaning [i.e. reference] of the concept-word *A*' is as problematic as 'the concept *Φ*', since 'the definite article before "meaning" points to an object' (1892–5, p. 122). But he now thinks that the difficulty is avoidable, even in natural language:

> it would be better to confine ourselves to saying 'what the concept-word *A* means', for this at any rate is to be used predicatively: 'Jesus is what the concept-word "man" means' in the sense of 'Jesus is a man'. (p. 122)

What is important here is what Frege does not say. He does *not* object to the definite article before 'concept-word', either in the illegitimate 'the meaning of the concept-word *A*' or in its bona fide replacement 'what the concept-word *A* means'. But the definite article is only in order if concept-words are themselves objects, and not functions as Hugly and Geach contend.

Dummett's reading of the footnote is completely different from Hugly's and Geach's. For him, it contains an error, not an insight: 'Frege was quite wrong in pretending that the same ills affect the formal mode of speech' (1955, p. 75). He supposes that the footnote is ambiguous. Frege may have intended assimilating his paradoxical 'the concept *horse* is not a concept' either to 'the predicate "is red" is not a predicate' (the more appropriate analogue) or to ' "the predicate 'is red' " is not a predicate' (the sentence suggested by his actual words). But Dummett finds neither of them paradoxical: the first is straightforwardly false, the second straightforwardly true. However the ambiguity is resolved, then, the comparison between material and formal modes is void.

Dummett is right about the truth-values of the two sentences in the formal mode, but wrong to ascribe error and the offensive 'pretending'. Frege was surely not deluded. The unparadoxical truth-values follow immediately from his own view of the matter. And it goes against all we know of his writing—its strident and compelling honesty— to suppose that he was out to delude his readers.

These commentators have all missed the point of comparison intended in the footnote. They read Frege as meaning to assimilate *sentences* in material and formal modes, whereas he is actually assimilating the *proper names* 'the concept *horse*' and 'the grammatical predicate "is red" ', while at the same time contrasting them both

with 'the city of Berlin' and 'the volcano Vesuvius', a contrast that he mentions in the relevant passage of the main text. The assimilation and contrast do not turn on the failure or success of intended and expected reference. For while each of the second pair hits the target, it is not the case that each of the first pair misses. Unlike 'the concept *horse*', 'the grammatical predicate "is red" ' does refer to what one intends and expects, namely a predicate. Hence Frege must be pointing to a different contrast, as follows. Whereas there is nothing peculiar about the make-up of 'the city of Berlin' and 'the volcano Vesuvius', it is quite different with 'the concept *horse*':

> The peculiarity of our case is indicated by Kerry himself, by means of the quotation-marks around 'horse'; I use italics to the same end. There was no reason to mark out the words 'Berlin' and 'Vesuvius' in a similar way. (1892b, p. 186)

In fact, Kerry takes over Frege's own use of quotation marks in *Grundlagen* (1884, §46: 'the concept "horse that draws the King's carriage" '), while in the German original of 'On Concept and Object' Frege had used expanded spacing between characters, as in 'the concept h o r s e'. Some such device is useful, since in the plain 'the concept horse', the doubling up of common nouns is hard to fathom, while 'the concept is a horse' is worse, since it is naturally read as a sentential clause. According to Frege's account, however, such a device is not just an aid to intelligibility; it also helps to create a context that shifts syntactic and semantic categories. He says that 'horse' is a predicate standing for a concept (in his informal writings, Frege commonly omits the copula and article, and sometimes fails to indicate empty places). But when the expression is italicised and given an appositional prefix as in 'the

concept *horse*', the result is a proper name standing for an object. No such shift

occurs between 'Berlin' and 'the city of Berlin', or between 'Vesuvius' and 'the

volcano Vesuvius'. This is the contrast that Frege intends.

The function of Frege's footnote is now clear. Although he was concerned to

emphasise the peculiarity of 'the concept *horse*', naturally he searched for more

commonplace phrases of similar construction in order to rebut the charge of special

pleading. The reference-shifting contexts of 'On Sense and Meaning' were at the front

of this mind. One of them—direct quotation of expressions—gave him the

comparison that he desired. Just as italics help turn the predicate 'horse' into the

proper name 'the concept *horse*', so quotation marks help turn the predicate 'is red'

into the proper name 'the grammatical predicate "is red" '. The final sentence of the

footnote—'by the very act of calling it a predicate, we deprive it of this property'—is

not intended to make the paradoxical claim that the predicate 'is red' is not a

predicate. That would be to place the 'it' at the level of things mentioned, whereas

Frege means to be describing the expression used. In other words, the 'it' is the

expression 'is red', which normally functions as a predicate, but does not do so when

it occurs within the context 'the grammatical predicate "is red" '.

In the footnote Frege is not at his lucid best. He does not take care to signal

explicitly the comparison that he has in mind. Nor does he help by concluding it with

an ambiguous sentence. But to read it as untypically sloppy is better than wrongly

imputing error (Dummett), and better than ascribing a view of predicates as linguistic

functions (Hugly and Geach) which goes against everything else Frege said, even in

the very same paper.

This reading of the footnote is also consonant with Frege's later remarks in a letter

to Russell. He once more faces the paradox of the concept *horse*, now in the more

general form of the difficulty in saying of functions that they are functions: 'the nature

of language forces us to make use of imprecise expressions … "*A* is a function" is

such an expression: it is always imprecise; for "*A*" stands for a proper name' (1902, p.

136). He points out that there is no such difficulty at the linguistic level:

> Instead of using the imprecise expression 'ξ is a function', we can say:
>
> ' "( ).3 + 4" is a function-name'. (p. 136)

But if '*A* is a function-name' is precise, function-names can be named by proper

names that take the place of '*A*'. And this is exactly what he says of his example:

> ' "( ).3 + 4" ' is a proper name, and its meaning is the function name '( ).3 +
>
> 4'. (p. 136)

If follows that Frege regards function-names as *objects*, since only objects can be

named by proper names.

2.4 Last thoughts

In his last published work, 'Compound Thoughts', Frege appears on a cursory

inspection to deny that function-names are really unsaturated, and to deprive them of

empty places. Was this an abrupt about-turn? It needs investigation. First, the context.

His attention had turned to the intermediate realm of sense, in particular to *thoughts*,

the senses of sentences. Although he had given up thinking of arguments and

functions as parts of the corresponding values at the level of reference, he continued

to transfer the relation of part to whole from sentences to the thoughts they express, and applied the idea that 'combination into a whole always comes about by the saturation of something unsaturated' (1923, p. 390) to these whole thoughts. In the paper in question, he investigates 'connectives', by which he means the kind of sense that joins several thoughts into one compound thought. Since the thoughts that are compounded are already saturated wholes, connectives must themselves be unsaturated in order to produce a saturated compound thought. He begins with the function-name 'and', which

> seems doubly unsaturated: to saturate it we require both a sentence preceding and another following. And what corresponds to 'and' in the realm of sense must also be doubly unsaturated: inasmuch as it is saturated by thoughts, it combines them together. As a mere thing, of course, the group of letters 'and' is not more unsaturated than any other thing. It may be called unsaturated in respect of its employment as a symbol meant to express a sense, for here it can have the intended sense only when situated between two sentences: its purpose as a symbol requires completion by a preceding and a succeeding sentence. It is really in the realm of sense that unsaturatedness is found, and it is transferred from there to the symbol. (p. 393)

Appearances to the contrary, Frege is not in fact denying that the expression 'and' is unsaturated. He is only insisting that when it is so described it must be coupled with a sense. For if it is considered 'as a mere thing' it is divorced from any sense, and consequently there is nothing that could determine that it is subject to a grammatical rule of combination. But when it is understood as expressing a sense, the sense

dictates that it 'requires completion by a preceding and a succeeding sentence'. (The distinction between thing and symbol also features in his celebrated discussion of identity (1891), pp. 157–8; the notion of an expression 'as a mere thing' is Dummett's attenuated notion of expression discussed in §1.3).

   But what of empty places? Frege does not mention them in this passage. It might therefore be thought that unsaturatedness as it pertains to function-names no longer has to do with empty places, but is now merely a way of describing the grammatical rules that govern expressions. Turn the page, however, and empty places or 'gaps' appear repeatedly. For example, in the discussion of his fourth kind of compound thought, Frege says

> The connective is the sense of whatever occurs in 'A or B' apart from 'A' and B', that is, the sense of
>
>   '(or   )'
>
> where the gaps on both sides of 'or' indicates the twofold unsaturatedness in the connective. (p. 396)

Frege continued equipping function-names with empty places to the end of his working life; see his unpublished (1924/5), p. 272, and the related letter (1925), p. 55.

2.5 Must predicates have empty places?

Frege's expressions with empty places are legitimate candidates for predicates. They are clearly distinct from the other five that I have discussed, viz. plain expressions, schemata, linguistic functions, properties of sentences, and properties of (or relations between) terms. But he does not claim that his candidate is merely one among many.

On the contrary, he repeatedly implies that his way of regarding them is mandatory. Since he simply takes it for granted that function-names are expressions of some kind or other, he fails to argue against candidates that fail to be any kind of expression. He does argue, however, against regarding function-names as expressions of a kind different from his own. Against schemata, he says:

> when we say 'the function $1 + \xi - \xi$', the letter 'ξ' is not part of the function-sign; for the proper name '$1 + 3 - 3$' is composed of the function-name and the proper name '3', and the letter 'ξ' does not occur in it at all. (1914, p. 239)

He is assuming that a function-name occurs in a more complex expression through being a part of it. Thus the function-name cannot contain a Greek letter, since the letter itself does not figure in the more complex expression in which the function-name occurs. Here he makes the same mistake as Geach (see §1.2a). He fails to allow for the different sense of 'occur', according to which the schema '$1 + \xi - \xi$' can quite properly be said to occur in '$1 + 3 - 3$' even if it is not a part of it.

In a single note Frege takes his style of argument one step further, distinguishing a function-name as it occurs on its own from a function-name as it occurs in combination. He says that 'sin ( )' (with empty brackets merely indicating the empty place) is 'meant only for the exceptional case where we want to symbolize a function is isolation. In "sin 2", "sin" by itself already symbolizes the function' (1904a, p. 291, fn. 3). In other words, a function-name in isolation is an expression with an empty place, but in combination it reduces to a plain expression. Evidently, he reasons that the function-name 'sin ( )' is not a part of 'sin 2' on the ground that the latter does not feature an *empty* place. He duplicates function-names, then, by using a narrow notion

of part as it relates to complex expressions. Yet the notion is in fact quite elastic. There is a perfectly good sense of 'part' in which an expression with an empty place is a part of a more complex expression even though its place is then filled (think of the parts of a completed jigsaw puzzle). Indeed, Frege himself relied on this notion of part in his other explanations of the make-up of complex expressions. As to construction, he says 'If we call the parts of the sentence that show gaps unsaturated and the other parts complete, then we can think of a sentence as arising from saturating an unsaturated part with a complete part' (1906, p. 201). As to decomposition, he says that a sentence 'may be imagined to be split up into two parts; one complete in itself, and the other in need of supplementation, or "unsaturated" … it contains an empty place' (1891, p. 146).

In fact, for the purposes of describing the construction and decomposition of sentences, it is quite unnecessary to characterise function-names as parts of those sentences at all. Rather than describing '*Fa*' as formed by *filling* the empty place in '*F* ', we can say instead that it is formed by *replacing* 'ξ' in the schema '*Fξ*', even though the schema is not part of the result. Likewise for decomposition, but in reverse. On this score, then, there is nothing to choose between schemata and Frege's own expressions with empty places as candidates for function-names. The same goes for plain expressions. The sentence '*Fa*' can equally well be described as formed by *attaching* the plain '*F*' to the term '*a*', while '*F*' itself is the result of the converse operation. In each of these three accounts, the function-name itself is a raw material for the construction, and a product of the decomposition. If function-names are regarded as things other than expressions, any of these accounts may still be given, but one will now use 'auxiliary expressions' (Noonan's phrase from §1.3) as the raw materials and products, rather than the function-names themselves.

Did Frege have any other reason for equipping function-names with empty places, rather than regarding them as plain expressions? He claims that without an empty place 'the function-name becomes a meaningless proper name' (1904b, p. 161). But building in an empty place is *not necessary* to make an expression a function-name and to prevent it from becoming a proper name. He is wrong to think that a proper name and function-name must 'differ essentially' (1904b, p. 161), if by this he means that they are intrinsically different types of thing. Both can be regarded in the same way as plain expressions, since it is enough that they differ in their relational properties. Their grammatical behaviour is different, they express different types of sense, and they stand for different types of referent. In passing, it is worth noting that Frege's empty places are *not sufficient* by themselves to mark out an expression as a function-name. At the end of §2.3, we saw that the function-name '(   ).3 + 4' is named by the proper name ' "(   ).3 + 4" '. His brief remarks on quotation (1892a, pp. 159, 165) suggest that enclosure by quotation marks creates a context in which the enclosed expression shifts reference. It follows that the expression with an empty place

   (   ).3 + 4

does double duty. Outside of quotation marks, it serves as a function-name. But when placed between them, it serves as a proper name, in which case its empty place alone cannot mark it out as a function-name. His thesis that nothing can be both a proper name and a function-name must therefore be qualified by excluding such shifts in context.

Frege's talk of completion or saturation of incomplete expressions by complete ones indicates that empty places are supposed to play another role, namely in explaining how a sentence differs from a mere list. For example, a string of proper names is not a

sentence. The proper names 'hold aloof from one another … however we put them together, we get no sentence' (1892b, p. 193), whereas

> Concept words and proper names are exactly fitted for one another. Because of its need for completion (unsaturatedness, predicative nature), a concept word is unsaturated, i.e. it contains a gap which is intended to receive a proper name. Through such saturation or completion there arises a sentence (1925, p. 55, with 'sentence' replacing the translator's 'proposition').

There are *two* contrasts in this area that may be thought to demand explanation. The first is the contrast between the one sentence and the many items in a string of proper names. The sentence is made of many items, but what makes them into one thing, while the string remains merely many? One answer is saturation.

It is disputable whether a string of proper names *is* its many items, for it may well be regarded instead as a single expression *made* from its many items. After all, it is *a* string. If so, saturation is not necessary to explain how one thing is made from many, since the string is such a thing, but does not feature saturation. Concatenation is enough.

If one resists treating such a string as a single expression, consider instead a word. It is one thing made from many letters, but even Frege did not suppose that some letters are saturated by others (which ones?). Or again, think of a single, semantically simple proper name consisting of several words, such as 'New York' or 'The Big Apple'.

The second contrast is independent of the first. Even if we regard a string of proper names as a single expression, it still does not count as a sentence. So what makes one expression a sentence and another not? Frege invokes saturation to explain the

difference (it has to be saturation of a particular kind, since saturation of other kinds features in expressions other than sentences, e.g '2+3'). But saturation is again not necessary for the job. If an account is demanded why a particular expression is or is not a sentence, general rules of grammatical combination may be given from which an answer can be deduced (at least in the case of a formal language). Contra Frege, there is no need to think that the different rules governing function-names and proper names have to be reflected in different intrinsic properties, the one kind having empty places, the other not. Nor can this kind of intrinsic difference be made to take over the work of grammatical rules, since ungrammatical combinations can easily be written down, e.g. '$\Phi(\ ) = X(\ )$' violates the rule that only proper names may flank '=' (*pace* Stenius, who thinks that Frege's account of function-names shows the '"superfluousness" of a theory of types', i.e. he supposes that grammatically impermissible combinations are impossible (1976, p. 79)).

Frege's apparatus of empty places does indeed make the failure of grammaticality visually apparent: the empty places are plainly not filled. This is hardly an advantage of his notation, however, since in e.g. standard presentations of the predicate calculus one can effectively decide whether a particular predicate occurs in a well-formed combination on the basis of its font, case or other typographical features, together with similar features of the other expressions with which it is combined, and general rules specifying permissible combinations. It is not just an effectively decidable matter, it is easy to determine. No empty places are needed; we have managed well without them.

A final argument moves from the distinctive nature of functions to the need for empty places in the corresponding function-names:

what is distinctive of a function, as compared with an object, is precisely its
'unsaturatedness', its needing to be completed by an argument; and this
feature must also come out in the symbolism. (1903, §147, note)

I can accept that functions need to be completed by arguments if this means that it is
in the nature of functions to be applied to arguments (though not in any sense having
to do with parts and wholes). As for the corresponding function-names, one can say
that they need to be completed by argument-terms, in the sense that their grammar
dictates that function-names can only properly occur in larger contexts when so
combined. They are governed by this kind of grammatical rule, since they are meant
to stand for functions that need completion. In this sense, the nature of functions does
come out in the symbolism. But it does not follow that function-names need to be
equipped with empty places. It is enough that they have the grammar they do; they
can look like anything you please. Again, the conclusion is the same: there is nothing
to choose between plain expressions and expressions with empty places. Although I
have concentrated in the previous few paragraphs on the alleged competition between
these two rival candidates, what I have said can easily be adapted to show that any of
the other candidates for function-names can do as well as Frege's own (as before, for
some purposes one will need to invoke auxiliary expressions rather than the chosen
candidates for function-names themselves).

Frege was the original arch-prescriptivist about logic. He claimed that natural
language is replete with faults that are to be corrected in his ideal, symbolic language.
For example, proper names *should* stand for one thing and one thing only (i.e. they
should neither be empty nor plural); function-names *should* be neither vague nor
incompletely defined; sentences *should* be neither devoid of truth-value nor be

anything other than true or false. He voiced these prescriptions in the strongest terms, and held onto them with remarkable obstinacy. It went entirely against his grain to countenance alternatives. Although his arguments for his prescriptions are quite feeble, it has taken time for us to see his logical system for what it is: a brilliant development of one alternative among many. The paradigmatic status of the predicate calculus and its second-order extension shows that some of his prescriptions still retain their grip.

His view of the nature of function-names has some of the same characteristics: the strength of expression, the obstinate persistence, the criticism of natural language (now as 'covering up' the distinction between proper names and function-names), feeble supporting arguments. There is a striking difference, however. Unlike his prescriptions for logic, his candidate for function-names was never widely adopted. Indeed, as I have shown, his conception of them has been widely misunderstood. Different exegetes pin different, alien conceptions on him. Like him, they often argue that their chosen candidate for function-names is the right one, but in reality any will do. There is no competition and no uniquely right answer.

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