PHILOSOPHY TRIPOS Part IA

Tuesday 26 May 2015 09.00 – 12.00

Paper 3

LOGIC

Answer **three** *questions only; at least* **one** *from* **each** of **sections A and B.**

Write the number of the question at the beginning of each answer.

Each question has equal weight. A perfect answer would receive a notional 100 marks. For Section A (formal questions) the number in square brackets after each component of a question designates the number of marks that a full and correct answer to that component would merit.

STATIONERY REQUIREMENTS
20 Page Answer Book x 1
Rough Work Pad

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

SECTION A

1. (a) Show each of the following [40]:

(i) $P \vee Q, \neg Q \vee R, \neg P \rightarrow \neg R \vdash P$

(ii) $A \leftrightarrow \neg B \vdash (B \wedge \neg A) \vee (A \wedge \neg B)$

(iii) $(A \wedge B) \rightarrow \neg C, B \leftrightarrow C, (C \wedge A) \vee (\neg C \wedge \neg A) \vdash \neg A$

(iv) $\vdash (P \leftrightarrow Q) \rightarrow (\neg P \leftrightarrow \neg Q)$

(v) $\vdash (P \vee Q) \rightarrow [[(P \wedge Q) \vee (P \wedge \neg Q)] \vee (Q \wedge \neg P)]$

(b) Show each of the following [60]:

(i) $\exists y\, (Ryy \leftrightarrow G\mathrm{a}) \vdash \neg \forall x\, \neg(Rxx) \vee \neg G\mathrm{a}$

(ii) $\vdash \forall x\, [(\neg Fx \wedge Fx) \leftrightarrow (Gx \wedge \neg Gx)]$

(iii) $\exists x\, (Fx \wedge \forall y\, (Fy \rightarrow y = x)), F\mathrm{a} \vdash F\mathrm{b} \rightarrow \mathrm{a} = \mathrm{b}$

(iv) $\exists x Gx,\ \exists x\, [Fx \wedge \forall y\, (Fy \rightarrow y = x)], \forall y\, (Gy \rightarrow Fy) \vdash \exists x \forall y\, (Gy \leftrightarrow x = y)$

2. Answer all parts of this question.

(a) Using the following symbolisation key: [60]

domain: all living creatures
Bx: $\_\_\_\__x$ is a bulldog
Hx: $\_\_\_\__x$ is a human
Sx: $\_\_\_\__x$ skateboards
Axy: $\_\_\_\__x$ admires $\_\_\_\__y$
Bxy: $\_\_\_\__x$ is better at skateboarding than $\_\_\_\__y$
i: Tillman
r: Rodney

Symbolise the following English sentences as best you can in FOL, commenting on any difficulties or limitations you encounter:

i. Rodney is a human and Tillman is a bulldog, and both can skateboard.
ii. Everybody admires Tillman, because he is a bulldog who can skateboard.
iii. Tillman is the skateboarding bulldog.
iv. There are skateboarding bulldogs besides Tillman.
v. Some human and some bulldog skateboard exactly as well as each other.
vi. Tillman is better at skateboarding than some humans, but he is not better than Rodney.
vii. When it comes to skateboarding, Tillman is the best bulldog.
viii. No bulldog skateboards better than Tillman does.
ix. Tillman admires all skateboarding bulldogs, whether or not they are better at skateboarding than him.
x. Two humans who skateboard are better than Rodney at it.

(b) Show that each of the following claims is true [30]

i. $\forall x(Fx \leftrightarrow \exists y(\neg x = y \land Gy \land Rxy)), Fa, Fb, \neg a = b \nvDash \exists x\exists y\exists z(\neg x = y \land \neg x = z \land \neg y = z)$
ii. $\exists x\exists y\exists z(\neg x = y \land \neg y = z \land \neg x = z \land (Rxy \leftrightarrow Ryx) \land (Rxy \leftrightarrow Rxz)) \nvDash \forall x\forall y(Rxy \rightarrow Ryx)$
iii. $\forall x(x = a \lor x = b \lor x = c), \forall x((x = a \lor x = b) \leftrightarrow Fx), \forall x((x = b \lor x = c) \leftrightarrow Gx), Rab, \neg Rba \nvDash \exists x\exists y\exists z(\neg x = y \land \neg x = z \land \neg y = z)$
iv. $\exists x\neg\exists yRyx, \forall x\exists yRxy, \forall x\forall y\forall z((Rxy \land Ryz) \rightarrow Rxz) \nvDash \exists x\forall y\forall z((Rxy \land Rxz) \rightarrow y = z)$

(c) Explain why we insist that the domain of quantification for any interpretation in FOL must always contain at least one object. [10]

3. (a) Define the following notions from set theory: Cartesian product, Empty set, Power set, Proper subset [10]

(b) Give examples of the following: [40]

i. A set with exactly one subset.
ii. A set with exactly one proper subset.
iii. A set of which at least one member is also a subset.
iv. Two sets whose Cartesian product has exactly six members.
v. A set whose power set has the set containing the empty set as a member.
vi. A set whose intersection with its own powerset is not empty.
vii. A non-empty (and finite!) set whose members include all members of its own members.
viii. A set whose members are all sets.

(c) Define the following notions from the theory of relations: Ancestral, Converse, Equivalence relation, Equivalence class. [20]

(d) Each of the following statements is false. In each case give a counterexample, being careful to specify the domain for each answer. [30]

i. Every relation is its own ancestral.
ii. Every relation is its own converse.
iii. Every reflexive symmetric relation is transitive.
iv. Every symmetric transitive relation is reflexive.
v. Every reflexive transitive relation is symmetric.

4. (a) Write down the axioms of the probability calculus. Define conditional probability. State any form of Bayes's Theorem. [15]

(b) Condition C occurs in 0.01% of the population. Suppose that a test for C is developed: it is positive on 95% of occasions when C is present and on 1% of occasions when C is absent. You take the test and it is positive. What is the probability that you have C? [25]

(c) I draw two cards from a standard pack without replacement. Calculate the following probabilities [60]:

i. The probability that the second is a heart given that the first is a heart.
ii. The probability that the second is a heart given that the first is the queen of hearts.
iii. The probability that at least one of them is an ace.
iv. The probability that the second is at least as high as the first in the following ranking: 2-10, J, Q, K, A.

SECTION B

5. For a conditional to be true, does there need to be a connection of some kind between the antecedent and the consequent?

6. Is Grice's account of speaker's meaning successful? Can it be used as a basis for an account of sentence meaning?

7. Is it false that the present King of France is bald?

8. Are there any necessary truths that are not knowable a priori?

END OF PAPER