

PHILOSOPHY TRIPOS Part II

Monday 27th May 2013

13.30 – 16.30

Paper 7

MATHEMATICAL LOGIC

*Answer **three** questions only.*

Write the number of the question at the beginning of each answer. If you are answering the either/or question, indicate the letter as well.

STATIONERY REQUIREMENTS

20 Page Answer book x 1

Rough Work Pad

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

1. Outline a proof of the completeness of first-order logic without identity. Explain how the proof has to be modified to obtain a proof of the completeness of first-order logic with identity.
2. Outline a proof of the categoricity of second-order Peano Arithmetic. Does the result have any philosophical significance?
3. What is it for a logic to be decidable? Is there any reason to think that a logic ought to be decidable?
4. What is the axiom scheme of replacement? Should we believe it?
5. Outline an account of the ordinal numbers and their arithmetic. You should show that ordinal addition and ordinal multiplication are not commutative.
6. In what sense, if any, should we regard set theory as a foundation for mathematics?
7. 'The existence of non-standard models of first-order Peano Arithmetic is a technical result of no philosophical significance.' Discuss.
8. EITHER: a) Give a careful statement of Gödel's first incompleteness theorem. What support does it give to the view that minds are not machines?

OR: b) State Gödel's first incompleteness theorem and sketch a proof of it.
9. What was Hilbert's Programme? Was it refuted by Gödel's second incompleteness theorem?
10. 'Church's thesis links a formal to an informal notion and therefore cannot be proved.' Discuss.

END OF PAPER